Geometry Min. Competence Solutions.

① a $5x + y - 3 = 0$

$y = -5x + 3$

$m = -5$

$(4, -8)$

(a, b)

$y - b = m(x - a)$

$y - (-8) = -5(x - 4)$

$y + 8 = -5x + 20$

$y = -5x + 12$

② a Rhombus diagonals are perpendicular

$m_{BD} = 3$

$\Rightarrow m_{AC} = -\frac{1}{3}$

$(-3, 5)$

$a \quad b$

$y - b = m(x - a)$

$y - 5 = -\frac{1}{3}(x - (-3))$

$y - 5 = -\frac{1}{3}(x + 3)$

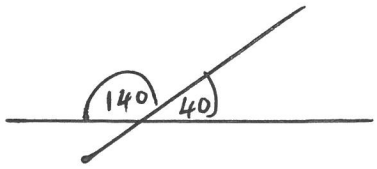
$\times 3 \qquad \times 3$

$3y - 15 = -1(x + 3)$

$3y - 15 = -x - 3$

$3y = -x + 12$

(3)(a)



$m = \tan\theta$

$= \tan 40$

$= 0.84$

$0.84 > 0.4$ so extreme skiing

(4)(a) Each circle has radius 4 units

$\Rightarrow$ Centre of other circle is $(4, 0)$

$(x-4)^2 + (y-0)^2 = 4^2$

$(x-4)^2 + y^2 = 16$

(5)(a) Simultaneous Equations

$(x+4)^2 + (y-2)^2 = 49$

$(x+4)^2 + ((x-1)-2)^2 = 49$

$(x+4)^2 + (x-3)^2 = 49$

$x^2 + 8x + 16 + x^2 - 6x + 9 = 49$

$2x^2 + 2x + 25 = 49$

$2x^2 + 2x - 24 = 0$

$(2x-6)(x+4) = 0$

$2x - 6 = 0$ $\quad$ $x + 4 = 0$

$2x = 6$ $\quad$ $x = -4$

$x = 3$

Line meets circle at 2 points where $x = 3$ + $x = -4$

$\Rightarrow$ Not a tangent.

(6)(a) $\overrightarrow{PT} = -\overrightarrow{PS} + \overrightarrow{ST}$

$$= -\begin{pmatrix} 3 \\ -8 \\ -6 \end{pmatrix} + \begin{pmatrix} -7 \\ 0 \\ 11 \end{pmatrix}$$

$$= \begin{pmatrix} -3 \\ 8 \\ 6 \end{pmatrix} + \begin{pmatrix} -7 \\ 0 \\ 11 \end{pmatrix} = \begin{pmatrix} -10 \\ 8 \\ 17 \end{pmatrix}$$

$$= \underline{-10i + 8j + 17k}$$

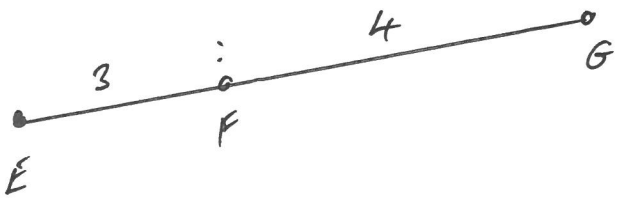
(7)(a) $\overrightarrow{PQ} = \underline{q} - p = \begin{pmatrix} 1 \\ 3 \\ -3 \end{pmatrix} - \begin{pmatrix} -1 \\ 4 \\ -8 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$

$\overrightarrow{QR} = \underline{r} - \underline{q} = \begin{pmatrix} 5 \\ 1 \\ 7 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ -3 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ 10 \end{pmatrix}$

$\overrightarrow{QR} = 2\overrightarrow{PQ} \Rightarrow$ 3 points are collinear and distance between flag 2 + 3 is twice the distance between flags 1 + 2.

$\Rightarrow$ Flags have been laid correctly.

(8)(a)



$$3\overrightarrow{FG} = 4\overrightarrow{EF}$$

$$3(\underline{g} - \underline{f}) = 4(\underline{f} - \underline{e})$$

$$3g - 3f = 4f - 4e$$

$$3g = 7f - 4e$$

$$7f = 3g + 4e$$

$$= 3\begin{pmatrix} 10 \\ 5 \\ -31 \end{pmatrix} + 4\begin{pmatrix} -11 \\ -2 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 30 \\ 15 \\ -93 \end{pmatrix} + \begin{pmatrix} -44 \\ -8 \\ 16 \end{pmatrix}$$

$$7\underline{f} = \begin{pmatrix} -14 \\ 7 \\ -77 \end{pmatrix}$$

$$f = \begin{pmatrix} -2 \\ 1 \\ -11 \end{pmatrix}$$

(9)(a) $\overrightarrow{DC} = \underline{c} - \underline{d}$

$= \begin{pmatrix} 20 \\ -7 \\ 7 \end{pmatrix} - \begin{pmatrix} 17 \\ -6 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$

$\overrightarrow{DB} = \underline{b} - \underline{d}$

$= \begin{pmatrix} 21 \\ -8 \\ 0 \end{pmatrix} - \begin{pmatrix} 17 \\ -6 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ -2 \\ -2 \end{pmatrix}$

$\overrightarrow{DC} \cdot \overrightarrow{DB} = (3 \times 4) + ((-1) \times (-2)) + (5 \times (-2))$

$= 12 + 2 + (-10)$

$= \underline{\underline{4}}$

$|\overrightarrow{DC}| = \sqrt{3^2 + (-1)^2 + 5^2}$

$= \sqrt{9 + 1 + 25}$

$= \underline{\underline{\sqrt{35}}}$

$|\overrightarrow{DB}| = \sqrt{4^2 + (-2)^2 + (-2)^2}$

$= \sqrt{16 + 4 + 4}$

$= \sqrt{24}$.

$\cos\theta = \dfrac{\overrightarrow{DC} \cdot \overrightarrow{DB}}{|\overrightarrow{DC}| . |\overrightarrow{DB}|} = \dfrac{4}{\sqrt{35} . \sqrt{24}}$

$= 0.138$

$\theta = \cos^{-1}(0.138)$

$= \underline{\underline{82.1^\circ}}$