CALCULUS MIN. COMPETENCE SOLUTIONS

1 (a)

$$f(x) = 5\sqrt{x} - \frac{2}{x^3}$$

$$= 5x^{\frac{1}{2}} - 2x^{-3}$$

$$f'(x) = \frac{5}{2}x^{-\frac{1}{2}} + 6x^{-4}$$

$$= \underline{\underline{\frac{5}{2x^{\frac{1}{2}}} + \frac{6}{x^4}}}$$

2 (a)

$$f(x) = 4\cos x$$

$$\underline{\underline{f'(x) = -4\sin x}}$$

3 (a)

$$y = 3x^2 + 2x + 2$$

$$\frac{dy}{dx} = 6x + 2$$

$$m_{x=-1} = 6(-1) + 2 = -6 + 2 = \underline{\underline{-4}}$$

When $x = -1$ $\quad y = 3(-1)^2 + 2(-1) + 2$

$$= 3 - 2 + 2 = 3$$

So point of contact is $(-1, 3)$

$$y - b = m(x - a)$$

$$y - 3 = -4(x - (-1))$$

$$y-3 = -4(x+1)$$
$$y-3 = -4x-4$$
$$\underline{\underline{y = -4x-1}}$$

(4) (a)

$$h = 40t - 2t^2$$

$$\frac{dh}{dt} = 40 - 4t$$

$$v = 40 - 4t$$

When $t = 10$, $v = 40 - 4(10)$
$= 0.$

$\underline{\underline{\text{Velocity} = 0}}$

(5) (a)

$$\int 5x^{3/2} + \frac{1}{x^3}\, dx$$

$$= \int 5x^{3/2} + x^{-3}$$

$$= \frac{5x^{5/2}}{5/2} + \frac{x^{-2}}{-2} + c$$

$$= \frac{2(5x^{5/2})}{5} - \frac{x^{-2}}{2} + c$$

$$= \underline{\underline{2x^{5/2} - \frac{1}{2x^2} + c}}$$

6 a

$$f'(x) = (x+3)^{-7}$$

$$f(x) = \int (x+3)^{-7}$$

$$= \frac{(x+3)^{-6}}{-6} + c$$

$$= -\frac{1}{6}(x+3)^{-6} + c$$

$$= \frac{-1}{6(x+3)^6} + c$$

7 a

$$\int 3\cos\theta = 3\sin\theta + c$$

8 a

$$\int_1^3 (x+1)^3 = \left[\frac{(x+1)^4}{4}\right]_1^3$$

$$= \left(\frac{(3+1)^4}{4} - \frac{(1+1)^4}{4}\right)$$

$$= \frac{4^4}{4} - \frac{2^4}{4} = 4^3 - \frac{16}{4}$$

$$= 64 - 4 = 60$$

sq. units

9 (a) $V(x) = 48x - \frac{1}{4}x^3$

Max when $V'(x) = 0$

$V'(x) = 48 - \frac{3}{4}x^2$

At max $48 - \frac{3}{4}x^2 = 0$

$\frac{3}{4}x^2 = 48$

$3x^2 = 192$

$x^2 = 64$

$x = \pm 8$

x	$\rightarrow$	-8	$\rightarrow$	8	$\rightarrow$
$V'(x)$	$-$	0	$+$	0	$-$
Shape	\	—	/	—	\

Max when $x = 8$

$V'(-10) = 48 - \frac{3}{4}(-10)^2$
$= 48 - 75$
negative.

$V'(0) = 48 - \frac{3}{4}(0)$
$= 48$
positive

$V'(10) = 48 - \frac{3}{4}(10)^2$
$= 48 - 75$
$=$ negative.

10 (a) $\int_1^3 x^2(3-x)$

$= \int_0^3 3x^2 - x^3$

$= \left[\frac{3x^3}{3} - \frac{x^4}{4}\right]_0^3$

$= \left[x^3 - \frac{1}{4}x^4\right]_0^3$

$= \left(3^3 - \frac{1}{4}(3)^4\right) - \left(0^3 - \frac{1}{4}(0)^4\right)$

$= \left(27 - \frac{81}{4}\right) - (0)$

$= \left(27 - 20\frac{1}{4}\right) -$

$= 6\frac{3}{4}$

$= \underline{\underline{6}}$

11 (a) Area $= \int_1^2 \text{Top} - \text{Bottom}$

$= \int_1^2 x - (x^2 - 2x + 2)$

$= \int_1^2 x - x^2 + 2x - 2$

$$= \int_1^2 \left(-x^2 + 3x - 2\right) \, dx$$

$$= \left[\frac{-x^3}{3} + \frac{3x^2}{2} - 2x \right]_1^2$$

$$= \left(-\frac{2^3}{3} + \frac{3(2)^2}{2} - 2(2) \right) - \left(\frac{-1^3}{3} + \frac{3(1)^2}{2} - 2(1) \right)$$

$$= \left(-\frac{8}{3} + 6 - 4 \right) - \left(\frac{-1}{3} + \frac{3}{2} - 2 \right)$$

$$= \left(-2\tfrac{2}{3} + 2 \right) - \left(\frac{7}{6} - 2 \right)$$

$$= -2\tfrac{2}{3} + 2 - \frac{7}{6} + 2$$

$$= 4 - 2\tfrac{2}{3} - \frac{7}{6}$$

$$= 1\tfrac{1}{3} - 1\tfrac{1}{6}$$

$$= \frac{1}{3} - \frac{1}{6} = \underline{\underline{\frac{1}{6}}}$$